

Topic 1: Linear motion and forces

1.1 Motion under constant acceleration

Science understanding

- Linear motion with constant velocity is described in terms of relationships between measurable scalar and vector quantities, including displacement, distance, speed, and velocity
 - Solve problems using $v = \frac{s}{t}$
 - Interpret solutions to problems in a variety of contexts.
 - Explain and solve problems involving the instantaneous velocity of an object.
- Acceleration is a change in motion. Uniformly accelerated motion is described in terms of relationships between measurable scalar and vector quantities, including displacement, speed, velocity, and acceleration.
 - Solve problems using equations for constant acceleration and $a = \frac{\Delta v}{\Delta t}$.
 - Interpret solutions to problems in a variety of contexts.
 - Make reasonable and appropriate estimations of physical quantities in a variety of contexts.
- Graphical representations can be used qualitatively and quantitatively to describe and predict aspects of linear motion.
 - Use graphical methods to represent linear motion, including the construction of graphs showing:
 - position *versus* time
 - velocity *versus* time
 - acceleration *versus* time.
 - Use graphical representations to determine quantities such as position, displacement, distance, velocity, and acceleration.
 - Use graphical techniques to calculate the instantaneous velocity and instantaneous acceleration of an object.
- Equations of motion quantitatively describe and predict aspects of linear motion.
 - Solve and interpret problems using the equations of motion:

$$v = v_0 + at$$

$$s = v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2as$$
- Vertical motion is analysed by assuming that the acceleration due to gravity is constant near Earth's surface.
- The constant acceleration due to gravity near the surface of the Earth is approximately $g = 9.80 \text{ ms}^{-2}$.
 - Solve problems for objects undergoing vertical motion because of the acceleration due to gravity in the absence of air resistance.
 - Explain the concept of free-falling objects and the conditions under which free-falling motion may be approximated.
 - Describe qualitatively the effects that air resistance has on vertical motion.
- Use equations of motion and graphical representations to determine the acceleration due to gravity.

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Speed

The speed of an object is defined as the distance the object travels per unit time.

The distance travelled by an object is simply how far it moves.

The unit of speed is kilometre per hour (kmh^{-1}) or metre per second (ms^{-1}).

Note: We write kmh^{-1} not km/h and ms^{-1} not m/s .

The standard international (SI) unit is ms^{-1} .

The equation for calculating speed is $speed = \frac{distance}{time}$

Using symbols we write $v = \frac{s}{t}$

Where v = speed, s = distance and t = time.

If the distance is in kilometres and the time is in hours, the unit of speed is kmh^{-1} .

If the distance is in metres and the time is in seconds, the unit of speed is ms^{-1} .

Different types of speed

Average speed

Calculating the speed of an object often involves calculating an average speed. Average speed does not take into account any changes in motion. It involves the total distance travelled and the total time. It doesn't indicate whether the object speeds up, slows down or stops during the journey. For instance, a car may travel between two towns. It speeds up as it takes off from a set of lights, it slows down as it approaches the next set of lights and it temporarily stops when it reaches a red light.

$$v_{av} = \frac{s_{total}}{t_{total}}$$

Constant speed

Constant speed means that an object travels exactly the same distance every unit of time. Light travels with a constant speed of 3×10^8 metres every second. Sound waves travel with a constant speed of 330 metres every second in air (this can change depending on the density of the air). If a car is travelling with a constant speed of 60 kmh^{-1} , this means that it travels **exactly** 60 kilometres every hour.



The diagram above illustrates constant motion or speed. The dots are equally spaced, which means the object travels the same distance per unit of time, i.e. it travels with constant speed.

Instantaneous speed

Instantaneous speed is the speed of an object at a particular instant in time. It is what the speedometer in a car measures. As the car speeds up or slows down the needle on the speedometer points to the speed of the car at a particular instant of time.



Science as a human endeavour

Laser guns

Laser guns work by sending out pulses of infra-red laser light towards a moving object, such as a car. The time taken for a pulse to return to the gun is recorded. The distance to the car is calculated using:

$$s = vt = 3 \times 10^8 \times \frac{t_1}{2}$$

The object continues moving, and the time taken for a second pulse to return to the laser gun is recorded. The new distance to the car is calculated using:

$$s = vt = 3 \times 10^8 \times \frac{t_2}{2}$$

The distance travelled by the car between the two pulses is the difference between these two values. The speed of the car is calculated using:

$$v = \frac{s_{travelled\ between\ pulses}}{t_{between\ pulses}}$$

Investigate other ways of calculating the speed of an object, e.g. radar gun, point-to-point cameras. What are the benefits and limitations?

Running with dinosaurs

How did Robert Alexander (1976) develop a method for determining the gait and speed of dinosaurs?

Common Conversions useful to problem solving

km to m	$\times 1000$ or 10^3
cm to m	$\div 100$ or $\times 10^{-2}$
minutes to seconds (s)	$\times 60$
hours to s	$\times 60 \times 60$
days to s	$\times 24 \times 60 \times 60$
kmh^{-1} to ms^{-1}	$\div 3.6$
ms^{-1} to kmh^{-1}	$\times 3.6$

Worked examples

1. A dog runs 30 m in 4.0 s. Calculate the average speed of the dog.

$$v = \frac{s}{t} = \frac{30}{4} = 7.5 \text{ ms}^{-1}$$

2. A marble circles the inside rim of a bowl of radius 15.0 cm five times in 20.0 s.

Determine the average speed of the marble.

$$\text{radius} = 15.0 \text{ cm} = 0.15 \text{ m}$$

(The distance covered is the circumference of the bowl. We calculate the circumference using $2\pi r$)

$$t = 20.0 \text{ s}$$

$$v = \frac{s}{t} = \frac{2\pi r \times 5}{20} = \frac{2\pi \times 0.15 \times 5}{20} = 0.236 \text{ ms}^{-1}$$

3. A boat travels 10.0 km in 30.0 minutes.

- (a) Calculate the average speed of the boat in kmh^{-1} and ms^{-1} .

$$s = 10.0 \text{ km}$$

$$t = 30.0 \text{ minutes} = 30 \div 60 = 0.5 \text{ h}$$

$$v = \frac{s}{t} = \frac{10}{0.5} = 20.0 \text{ kmh}^{-1}$$

$$20 \text{ kmh}^{-1} = 20 \div 3.6 = 5.56 \text{ ms}^{-1}$$

- (b) Calculate the distance travelled by the boat in 6.50 hours.

$$s = vt = 20 \times 6.5 = 130 \text{ km}$$

4. Light travels with a speed of $3.00 \times 10^8 \text{ ms}^{-1}$. Calculate the time taken for light to travel from the Sun to Earth, a distance of $1.50 \times 10^{11} \text{ m}$.

$$t = \frac{s}{v} = \frac{1.5 \times 10^{11}}{3 \times 10^8} = 500 \text{ s}$$

Vector and scalar quantities

Quantities that have **size or magnitude only** are called **scalar** quantities. Examples include mass, time, energy and temperature.

Quantities that have both **magnitude and direction** are called **vector** quantities. One example is force (a push or a pull). This is because an object can be pulled or pushed in a given direction e.g. 5 N east.

We will come across many vector quantities throughout this course. We will deal with each as it arises. Some examples of scalar and vector quantities are summarised in the table below.

Scalar quantities	Vector quantities
distance	displacement
speed	velocity
time	acceleration
mass	force
volume	momentum
temperature	electric field
charge	magnetic field
heat	
energy	
power	

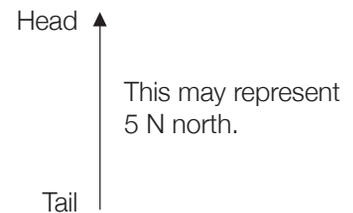
Representing vector quantities

From your Year 10 studies, you will be familiar with a force being a push or pull. Force has magnitude and direction, and is therefore a vector quantity.

A vector quantity is denoted in **bold type** or with an arrow above the symbol.

$$\mathbf{F} = 5 \text{ N} \quad \text{or} \quad \vec{F} = 5 \text{ N}$$

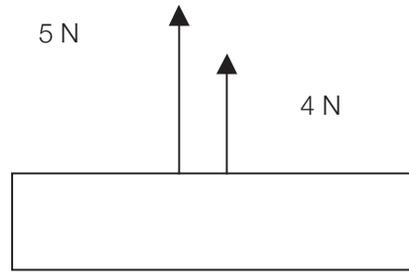
An arrow is used to represent the vector quantity. The **length** of the arrow represents the **magnitude** of the vector and the **arrow** head points in the **direction** of the vector.



Adding vector quantities

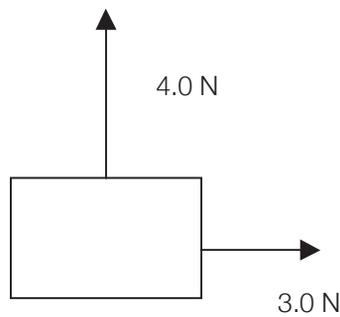
Worked examples

5 N north + 4 N north



9 N north

4 N north + 3 N east



The total or resultant force is $F_R = 4 \uparrow + 3 \rightarrow$

A **vector triangle** is drawn in order to 'add' the two vectors. The vectors are added 'head to tail'. The order doesn't matter.

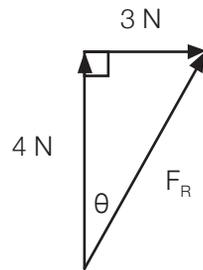
Pythagoras' Theorem is used to find the **magnitude of the** resultant force and **trigonometric ratios** are used to find the **direction**.

$$F_R = \sqrt{3^2 + 4^2} = 5N$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta = 37^\circ$$



The final answer is expressed with magnitude and direction.

$$F_R = 5.0 \text{ N } 37^\circ$$

Note: A scale diagram could have been used to solve the above problem.

Displacement and velocity

Distance is how far an object has travelled (or length covered). It is a **scalar** quantity because no direction is involved.

Position is the location of a body.

Displacement is the change in position and includes direction. It is a **vector** quantity since it involves both magnitude (size) and direction.

Velocity is defined as displacement per unit time. Velocity is a **vector** quantity.

Worked examples

1. A man walks 5.0 km in a northerly direction and then 2.0 km in a southerly direction.

- (a) State the distance travelled by the man.

7.0 km

- (b) State the displacement of the man.

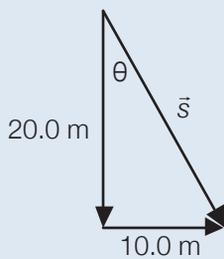
3.0 km North

2. A ferret races 20.0 m S and then 10.0 m east.

- (a) Calculate the distance travelled by the ferret.

30.0 m

- (b) Calculate the displacement of the ferret.



$$\vec{s} = \sqrt{20^2 + 10^2} = 22.4\text{m}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{20}$$

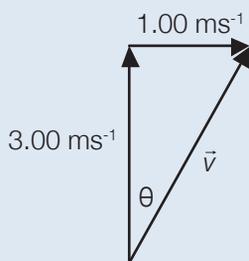
$$\theta = \tan^{-1}\left(\frac{10}{20}\right)$$

$$\theta = 26.6^\circ$$

$$\vec{s} = 22.4 \text{ m S}26.6^\circ\text{E}$$

3. A boat is rowed with a speed of 3.00 ms^{-1} in a northerly direction. It encounters a water current flowing at 1.00 ms^{-1} in an easterly direction.

- (a) Calculate the resultant velocity of the boat.



$$\vec{v} = \sqrt{3^2 + 1^2} = 3.16\text{ms}^{-1}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\theta = 18.4^\circ$$

$$\vec{v} = 3.16 \text{ ms}^{-1} \text{ N}18.4^\circ\text{E}$$

- (b) Calculate the boat's displacement after 10.0 minutes.

$$\vec{s} = \vec{v}t = 3.16 \times (10 \times 60) = 1896\text{m} = 1.90 \times 10^3 \text{ m N}18.4^\circ\text{E}$$

- (c) Assume that the rower's intention was to row to a destination directly north of his starting point. How far off course is the boat after 10.0 minutes?

$$s = vt = 1 \times (10 \times 60) = 600\text{m}$$

- (d) How could the effect of the current be compensated for?

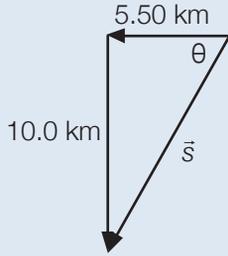
Row into the current with a velocity of $3.16 \text{ ms}^{-1} \text{ N}18.4^\circ\text{W}$

4. A man for his morning fitness routine walks 5.50 km W and then turns and walks 10.0 km S in 3.00 hours and 15.0 minutes. Calculate the

(a) distance travelled by the man.

$$15.5 \text{ km} = 16.0 \text{ km}$$

(b) man's final displacement.



$$\bar{s} = \sqrt{5.5^2 + 10^2} = 11.4 \text{ km}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{5.5}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.5}\right)$$

$$\theta = 61.2^\circ$$

$$\bar{s} = 11.4 \text{ km W}61.2^\circ\text{S}$$

(c) man's average speed for the journey.

$$v = \frac{s}{t} = \frac{15.5}{3.25} = 4.80 \text{ kmh}^{-1}$$

(d) man's average velocity for the journey.

$$\bar{v} = \frac{\bar{s}}{t} = \frac{11.4}{3.25} = 3.50 \text{ kmh}^{-1}$$

Subtracting vectors

If 5 N north is represented as 5N ↑, then -5N ↑ must mean 5N ↓ or 5 N south.

When subtracting a vector, it is added in reverse.

Therefore 5N north – 5 N south = 5N ↑ – 5N ↓ = 5N ↑ + 5N ↑ = 10N ↑

Examples

1. $50\text{N} \rightarrow -100\text{N} \leftarrow = 50\text{N} \rightarrow +100\text{N} \rightarrow = 150\text{N} \rightarrow$

2. $2\text{N} \downarrow -3\text{N} \downarrow = 2\text{N} \downarrow +3\text{N} \uparrow = 1\text{N} \uparrow$

Acceleration

If an object is not travelling with constant speed (i.e. it is speeding up or slowing down) is said to be accelerating.

Acceleration is the change in velocity per unit time or the rate of change in velocity.

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

where Δv = change in velocity

v_f = final velocity in ms^{-1}

v_i = initial velocity in ms^{-1}

Δt = time taken for the change in velocity in s

Units: ms^{-2}

Since velocity is a **vector** quantity, then the acceleration will also have a magnitude and direction and is therefore considered a **vector** quantity.

Notes:

1. If direction is not involved i.e. only the speed changes, then the acceleration of an object is the change in speed per unit time.
2. Although the speed of an object may be constant, a change in direction constitutes a change in velocity. The object is said to accelerate.
3. A constant acceleration of 3 ms^{-2} means that the object speeds up by 3 ms^{-1} every second i.e. the speed increases as follows after every second:
 $0 \text{ ms}^{-1}, 3 \text{ ms}^{-1}, 6 \text{ ms}^{-1}, 9 \text{ ms}^{-1}, 12 \text{ ms}^{-1}, 15 \text{ ms}^{-1} \dots$
4. If an object speeds up, the acceleration is **positive**.
5. If an object slows down, the acceleration is **negative**. This is sometimes called a deceleration.
6. The acceleration due to gravity is constant near the surface of the Earth and is $g = 9.80 \text{ ms}^{-2}$ towards the centre of the Earth.

Worked examples

1. A car accelerates from rest to a speed of 60.0 kmh^{-1} in 5.00 seconds. Calculate the acceleration of the car.

$$a = \frac{v_f - v_i}{\Delta t} = \frac{60 - 0}{5} = 3.33 \text{ ms}^{-2}$$

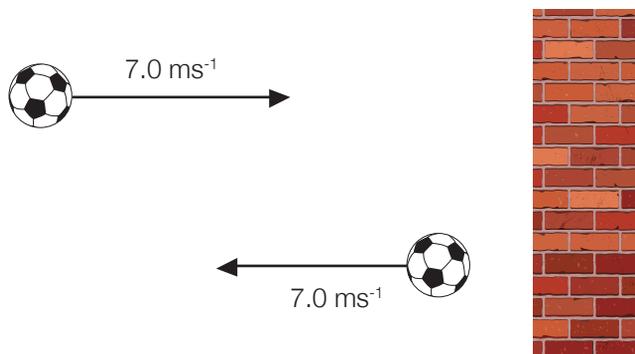
2. A truck can accelerate from rest at a rate of 4.00 ms^{-2} . Calculate its speed after 8.00 seconds. Answer in ms^{-1} and kmh^{-1} .

$$a = \frac{v_f - v_i}{\Delta t} \quad v_f = v_i + at = 0 + 4 \times 8 = 32.0 \text{ ms}^{-1} = 115 \text{ kmh}^{-1}$$

3. A ball is thrown vertically into the air with a speed of 10.0 ms^{-1} . Calculate the time taken to reach its maximum height.

$$a = \frac{v_f - v_i}{\Delta t} \quad \Delta t = \frac{v_f - v_i}{a} = \frac{0 - 10}{-9.8} = 1.02 \text{ s}$$

4. A ball collides with a wall as shown.



- (a) Calculate the ball's change in velocity.

$$v = v_f - v_i = 7 \leftarrow - 7 \rightarrow = 7 \leftarrow + 7 \leftarrow = 14 \text{ ms}^{-1} \leftarrow \text{ (90° away from the wall)}$$

- (b) If the collision takes 0.120 s, calculate the acceleration experienced by the ball.

$$a = \frac{\Delta v}{\Delta t} = \frac{14}{0.12} = 120 \text{ ms}^{-2} \leftarrow$$

**Helpful online resources**

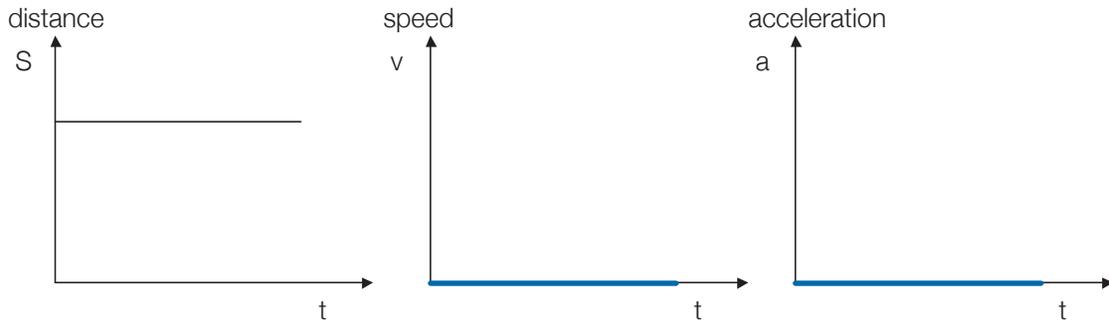
Explore the relationship between velocity and acceleration using the computer interactive 'The Maze Game'.

<https://phet.colorado.edu/en/simulation/legacy/maze-game>

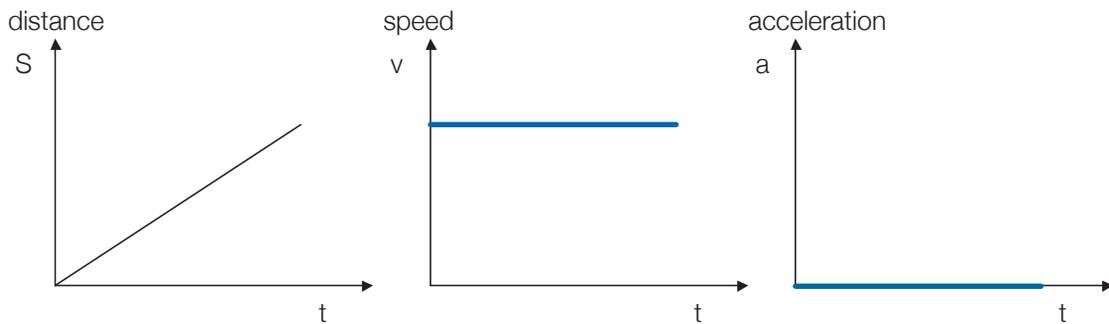


Graphing Motion

Stationary Motion



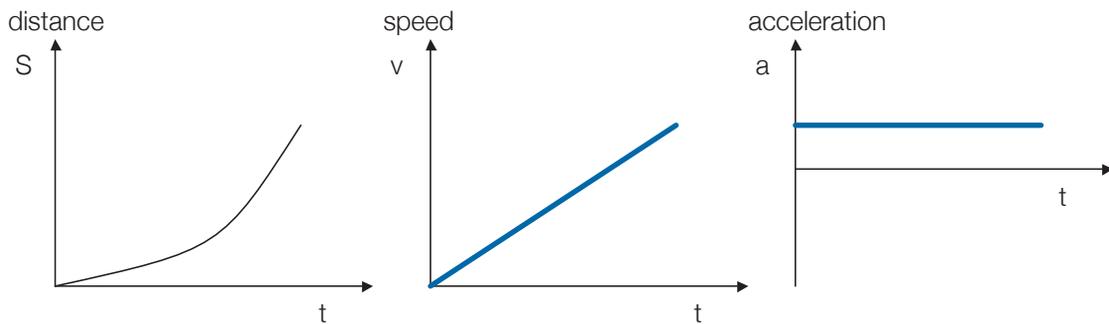
Constant Speed



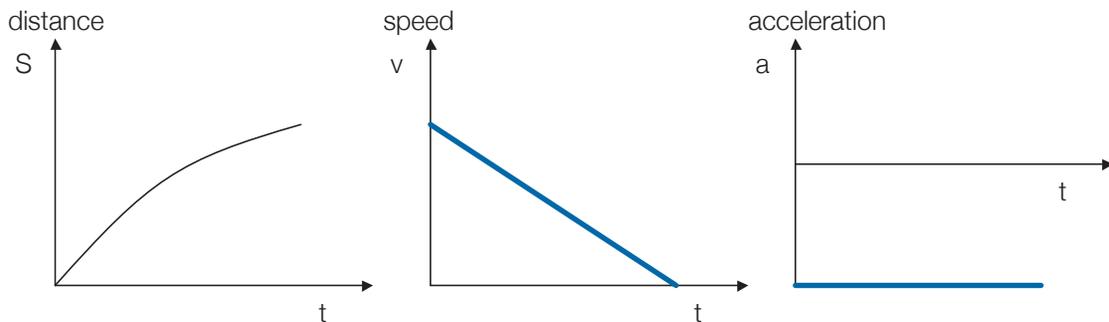
The **gradient** of a distance-time graph represents **speed**.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta s}{\Delta t}$$

Constant acceleration



The distance time graph above indicates that the distance travelled per unit time increases. This represents **accelerated** motion.



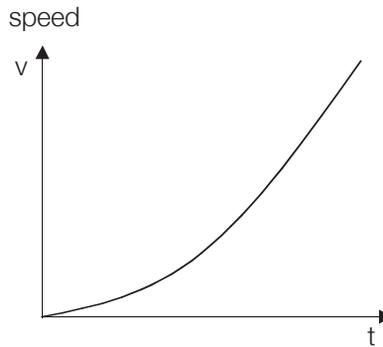
The distance time graph above represents a negative acceleration or **decelerated** motion as the distance travelled per unit time is decreasing.

The **gradient of the tangent** of a distance time graph at any particular time represents the **instantaneous velocity** at that time. We write

$$v = \frac{\Delta s}{\Delta t} \quad \text{as } \Delta t \rightarrow 0$$

The **area** under of a speed time graph represents **distance**.

Non constant acceleration



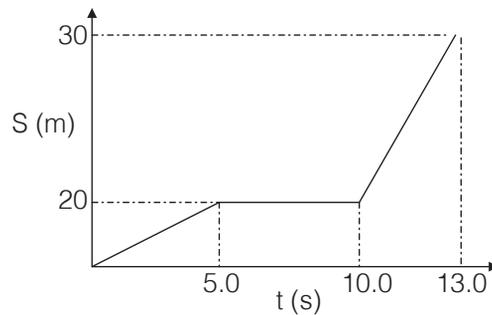
A curved speed time graph indicates that the speed is constantly changing.

The **gradient of the tangent** at a given point represents the **instantaneous acceleration**, i.e. the change in velocity that takes place over a very short period of time $\Delta t \rightarrow 0$. We write

$$a = \frac{\Delta v}{\Delta t} \quad \text{as } \Delta t \rightarrow 0$$

Worked examples

1. Consider the graph below for the motion of a toy cart.



Note: This diagram is not to scale

- (a) Describe the motion of the toy cart.

The toy cart travels with constant speed, travelling 20 m in 5 s. The cart then remains stationary for 5 s and then travels with a higher constant speed for the remaining 3 seconds.

- (b) State the total distance travelled by the toy car.

30 m

- (c) Calculate the average speed of the toy car.

$$v = \frac{s}{t} = \frac{30}{13} = 2.3 \text{ ms}^{-1}$$